



# Tracking Errors and Long-Run Performance of Leveraged ETFs

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## Research Objectives

Leveraged exchange-traded funds (ETFs) are relatively new to the world of investments but have become increasingly popular to aggressive investors. While a regular ETF tracks the value of a specific index of stocks, a leveraged ETF attempts to achieve a multiple of the return of the underlying index on a daily basis. This multiple can be positive in the case of bull ETFs or negative in the case of bear (or inverse) ETFs. To accomplish these objectives, leveraged and inverse funds pursue a range of investment strategies through the use of swaps, futures contracts, options and other derivative instruments. Due to the effect of compounding, operating expenses and daily resets, not to mention tracking errors, the performance of leveraged funds over longer periods of time can differ substantially from the performance (or inverse of the performance) of their underlying index or benchmark during the same period of time. Such performance deviations are often quite meaningful and unexpected over the long run.

This paper provides an empirical assessment of how well the leveraged ETFs track their underlying index. The results show that the tracking errors on average are small. However, substantial tracking errors do occur from time to time. Despite the price decay associated with leveraged ETFs, their long-run performance.

## Statistical Properties of Leveraged ETFs

### Arithmetic vs. Geometric Returns

When evaluating the performance of leveraged and inverse ETFs, we have to distinguish between geometric and arithmetic averages. The latter is the simple average of daily returns over a given period of time. The former, on the other hand, represents an equivalent but constant daily return over the same period.

Consider a time-series of daily returns of the underlying asset for T days:  $R_1, R_2, \dots, R_{T-1}$ , and  $R_T$ . The arithmetic daily average return,  $R_A$ , and the standard deviation of returns,  $\sigma$ , are defined as

$$R_A = \frac{1}{T} \sum_{i=1}^T R_i \quad \text{and} \quad \sigma^2 = \frac{1}{T} \sum_{i=1}^T (R_i - R_A)^2$$

The geometric mean,  $R_G$ , is defined as

$$R_G = -1 + \left[ \prod_{i=1}^T (1 + R_i) \right]^{1/T}$$

It can be shown that the following relationship holds between  $R_A$  and  $R_G$ :

$$\begin{aligned} R_G &\approx -1 + \prod_{i=1}^T \left( 1 + \frac{2}{T} R_i + \frac{2-T}{2T^2} R_i^2 \right) \\ &\approx \frac{1}{T} \sum_{i=1}^T R_i + \frac{1}{T^2} \sum_{i \neq j} R_i R_j + \frac{1-T}{2T^2} \sum_{i=1}^T R_i^2 \\ &= R_A - \sigma^2 / 2 \end{aligned}$$

### Price Decay of Leveraged ETFs

Now consider a leveraged bull ETF that aims to change in the same direction by n times the daily change of the underlying index. For simplicity, we assume no tracking error, management fees, or interest charges. Under this assumption, the arithmetic average of this nX ETF will be exactly  $nR_A$ , and the standard deviation will be  $n\sigma$ . It can be shown that its geometric average will be approximately  $nR_G - n(n-1) \sigma^2/2$

The last term,  $n(n-1) \sigma^2/2$ , is often referred to as the price decay that is caused by volatility. The summary for various leveraged ETFs is presented in the top table.

## Statistical Properties of Leveraged ETFs and the Underlying Index: A Summary

The underlying index (referred to as 1X) has an arithmetic average of  $R_A$ , geometric average of  $R_G$ , and standard deviation of  $\sigma$ . The leveraged and inverse ETFs are assumed to achieve exactly n times the return of the index on the daily basis, where n is positive for bull ETFs and negative for bear ETFs.

| Leveraged ETF | Description      | Arithmetic average | Geometric average              | Price Decay  |
|---------------|------------------|--------------------|--------------------------------|--------------|
| 1X            | Underlying index | $R_A$              | $R_G \approx R_A - \sigma^2/2$ | $\sigma^2/2$ |
| 2X            | 2X Bull ETF      | $2R_A$             | $2R_G - \sigma^2$              | $\sigma^2$   |
| 3X            | 3X Bull ETF      | $3R_A$             | $3R_G - 3\sigma^2$             | $3\sigma^2$  |
| -1X           | -1X Bear ETF     | $-R_A$             | $-R_G - \sigma^2$              | $\sigma^2$   |
| -2X           | -2X Bear ETF     | $-2R_A$            | $-2R_G - 3\sigma^2$            | $3\sigma^2$  |
| -3X           | -3X Bear ETF     | $-3R_A$            | $-3R_G - 6\sigma^2$            | $6\sigma^2$  |

## Data and Sample

This paper examines two groups of leveraged ETFs which are offered from ProShares, the pioneer of leveraged ETFs. The first panel in the following table presents the five ETFs using the S&P 500 index as the underlying index. The second panel lists the five ETFs that are based on the Russell 2000 index. The S&P500 index is essentially a large-cap index while the Russell 200 is a small-cap index. We collect daily returns for these indexes and ETFs from inception to year-end 2013.

| Name                                      | Symbol | Objective | Inception | Fee   |
|---|--------|-----------|-----------|-------|
| Panel A: LETFs based on S&P500 Index      |        |           |           |       |
| S&P500 EFT (benchmark)                    | SPY    | 1X        |           |       |
| Ultra S&P500                              | SSO    | 2X        | 6/19/2006 | 0.90% |
| UltraPro S&P500                           | UPRO   | 3X        | 6/23/2009 | 0.95% |
| Short S&P500                              | SH     | -1X       | 6/19/2006 | 0.90% |
| UltraShort S&P500                         | SDS    | -2X       | 7/11/2006 | 0.90% |
| UltraProShort S&P500                      | SPXU   | -3X       | 6/23/2009 | 0.93% |
| Panel B: LETFs based on Russell2000 Index |        |           |           |       |
| iShares Russell2000 (benchmark)           | IWM    | 1X        |           |       |
| Ultra Russell2000                         | UWM    | 2X        | 1/23/2007 | 0.98% |
| UltraPro Russell2000                      | URTY   | 3X        | 2/9/2010  | 0.98% |
| Short Russell2000                         | RWM    | -1X       | 1/23/2007 | 0.95% |
| UltraShort Russell2000                    | TWM    | -2X       | 1/23/2007 | 0.95% |
| UltraPro Short Russell2000                | SRTY   | -3X       | 2/9/2010  | 0.95% |

## Tracking Errors and Summary Statistics

The main objective of this study is examine how well the leveraged ETF (LETF) tracks its underlying index. For bull ETFs, tracking error is defined as ( $R_{LETF} - n R_{Index}$ ); For bear ETFs, tracking error is defined as ( $R_{LETF} + n R_{Index}$ ).  $R_{LETF}$  is the daily return of the LETF and  $R_{Index}$  is the daily return of the index, and n is the stated multiple.

For the S&P500-based LETFs, both the mean and median tracking errors are small. However, the minimum (-0.0667) and the maximum (0.06586) suggest the existence of substantial tracking errors.

| Name                   | Symbol | Daily Objective | Mean     | Median   | Min      | Max     | Standard Deviation |
|------------------------|--------|-----------------|----------|----------|----------|---------|--------------------|
| S&P500 EFT (benchmark) | SPY    | 1X              | /        | /        | /        | /       | /                  |
| Ultra S&P500           | SSO    | 2X              | -0.00014 | -0.00006 | -0.06607 | 0.03095 | 0.00357            |
| UltraPro S&P500        | UPRO   | 3X              | -0.00007 | -0.00002 | -0.03416 | 0.03153 | 0.00272            |
| Short S&P500           | SH     | -1X             | 0.00007  | -0.00002 | -0.02091 | 0.03691 | 0.00204            |
| UltraShort S&P500      | SDS    | -2X             | 0.00010  | 0.00005  | -0.04039 | 0.06586 | 0.00332            |
| UltraProShort S&P500   | SPXU   | -3X             | -0.00006 | -0.00007 | -0.04095 | 0.04037 | 0.00269            |

The results are similar for the Russell2000-based LETFs. Again, both the mean and median tracking errors are small. However, the minimum (-0.06161) and the maximum (0.05407) suggest the existence of substantial tracking errors.

| Name                            | Symbol | Daily Objective | Mean      | Median    | Min       | Max      | Standard Deviation |
|---------------------------------|--------|-----------------|-----------|-----------|-----------|----------|--------------------|
| iShares Russell2000 (benchmark) | IWM    | 1X              | /         | /         | /         | /        | /                  |
| Ultra Russell2000               | UWM    | 2X              | -0.000089 | 0.000000  | -0.049420 | 0.039220 | 0.003873           |
| UltraPro Russell2000            | URTY   | 3X              | -0.000049 | 0.000000  | -0.018610 | 0.015380 | 0.002433           |
| Short Russell2000               | RWM    | -1X             | -0.000011 | -0.000070 | -0.012470 | 0.012440 | 0.002125           |
| UltraShort Russell2000          | TWM    | -2X             | -0.000041 | -0.000065 | -0.036720 | 0.054070 | 0.003593           |
| UltraPro Short Russell2000      | SRTY   | -3X             | -0.000240 | -0.000140 | -0.061610 | 0.040540 | 0.003461           |

## Regression Analyses

Another approach to access the tracking effectiveness is regress the daily return of an EFT on the daily return of its underlying index as follows:

$R_{LETF} = \alpha + \beta R_{Index} + \epsilon$   
where  $R_{LETF}$  is the daily return of the LETF, and  $R_{Index}$  is the daily return of the index. The coefficient estimate,  $\beta$ , can be used to test the hypothesis that the leveraged ETF achieves its stated objective, i.e., n times the underlying index's return.

The regression results, presented in the following table, show that the adjusted  $R^2$  is very high and close to 1.0 in all regressions, indicating strong tracking effectiveness. Note that a  $R^2$  value of 1.0 indicates perfect tracking. Additionally, the estimated  $\beta$  is very close to its expected value.

| Name                                      | Symbol | Daily Objective | Expected $\beta$ | Estimated $\beta$ | Standard error | Adj. $R^2$ |
|---|--------|-----------------|------------------|-------------------|----------------|------------|
| Panel A: LETFs based on S&P500 Index      |        |                 |                  |                   |                |            |
| Ultra S&P500                              | SSO    | 2X              | 2.0              | 1.921             | 0.0055         | 0.985      |
| UltraPro S&P500                           | UPRO   | 3X              | 3.0              | 2.977             | 0.0076         | 0.996      |
| Short S&P500                              | SH     | -1X             | -1.0             | -0.9821           | 0.0033         | 0.979      |
| UltraShort S&P500                         | SDS    | -2X             | -2.0             |                   |                |            |
|   |        |                 |                  | -1.936            | 0.0052         | 0.987      |
| UltraProShort S&P500                      | SPXU   | -3X             | -3.0             |                   |                |            |
|   |        |                 |                  | -2.978            | 0.0075         | 0.993      |
| Panel B: LETFs based on Russell2000 Index |        |                 |                  |                   |                |            |
| Ultra Russell2000                         | UWM    | 2X              | 2.0              |                   |                |            |
|   |        |                 |                  | 2.002             | 0.0051         | 0.989      |
| UltraPro Russell2000                      | URTY   | 3X              | 3.0              |                   |                |            |
|   |        |                 |                  | 2.977             | 0.0076         | 0.997      |
| Short Russell2000                         | RWM    | -1X             | -1.0             |                   |                |            |
|   |        |                 |                  | -0.9996           | 0.0028         | 0.987      |
| UltraShort Russell2000                    | TWM    | -2X             | -2.0             |                   |                |            |
|   |        |                 |                  | -2.006            | 0.0047         | 0.990      |
| UltraPro Short Russell2000                | SRTY   | -3X             | -3.0             |                   |                |            |
|   |        |                 |                  | -2.955            | 0.0075         | 0.994      |

## Price Decay and Long-Run Performance

The statistical properties of LETFs suggests a price decay. Consider the case of the 2X bull ETF, which aims to achieve twice the daily return of the underlying index on a daily basis. Suppose the underlying index (i.e., the S&P 500 Index) has zero one-year total return. Due to the price decay caused by daily resets and volatility, the one-year total return for the bull ETF is -6.1%, even under perfect tracking.

The long-run performance is calculated as the cumulative daily return. The table below presents the year-end 2013 value of one dollar invested at the LETF at inception.

For instance, one dollar invested in the UltraPro S&P500 (3X) bull ETF on June 23, 2009 would become \$6.697 by the end of 2013, compared to only \$2.177 if invested in the benchmark index fund. However, the same dollar would be only \$0.041 if invested in the 3X bear ETF.

| Name                                      | Symbol | Daily Objective | Inception | Cumulative Return of LETF | Cumulative Return Of Benchmark index |
|---|--------|-----------------|-----------|---------------------------|--------------------------------------|
| Panel A: LETFs based on S&P500 Index      |        |                 |           |                           |                                      |
| Ultra S&P500                              | SSO    | 2X              | 6/19/2006 | 1.648                     | 1.729                                |
| UltraPro S&P500                           | UPRO   | 3X              | 6/23/2009 | 6.697                     | 2.199                                |
| Short S&P500                              | SH     | -1X             | 6/19/2006 | 0.452                     | 1.729                                |
| UltraShort S&P500                         | SDS    | -2X             | 7/11/2006 | 0.129                     | 1.743                                |
| UltraProShort S&P500                      | SPXU   | -3X             | 6/23/2009 | 0.041                     | 2.199                                |
| Panel B: LETFs based on Russell2000 Index |        |                 |           |                           |                                      |
| Ultra Russell2000                         | UWM    | 2X              | 1/23/2007 | 1.239                     | 1.628                                |
| UltraPro Russell2000                      | URTY   | 3X              | 2/9/2010  | 4.253                     | 2.024                                |
| Short Russell2000                         | RWM    | -1X             | 1/23/2007 | 0.355                     | 1.628                                |
| UltraShort Russell2000                    | TWM    | -2X             | 1/23/2007 | 0.060                     | 1.628                                |
| UltraPro Short Russell2000                | SRTY   | -3X             | 2/9/2010  |                           | 2.024                                |
|   |        |                 |           | 0.028                     |                                      |

## Summary

The study attempts to understand the statistical properties of leveraged and inverse ETFs. It also provides an empirical assessment of how well the leveraged ETFs track their underlying index. The results show that the tracking errors on average are small. However, substantial tracking errors do occur from time to time.

In conclusion, leveraged ETFs may be appropriate for aggressive investors who want to double or triple their short-term returns, but buy-and-hold investors must be warned of the long-run impacts of price decays.